GRAMA

Mathematica program for analytical calculations in ten-dimensional supergravity [1]

Version 1.0

User's Manual.

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Contents

1 Introduction

"GRAMA" is a MATHEMATICA package for doing symbolic tensor computations and complicated algebraic manipulations in ten-dimentional ($D = 10$), simple ($N = 1$) supergravity. The main new ingredients of this package inside the general MATHEMATICA invironment is the computation of complicated products of Dirac matrices and treatment of covariant derivatives: spinorial and vectorial.

Other *Mathematica* packages for high-energy physics and gravitation theory include: [1],[2],[3],[4],[5], [6],[7],[8],[9],[10].

The space-time dimension is introduced by the variable "dim". Analogously the dimension of spinorial representation is introduced by the variable "sdim". In our case of "dim" $= 10$ and we use 16-component description of spinors, i.e. "$sdim$" $= 16$. In principle with small modifications GRAMA may be applied for doing calculations in the four-dimensional supergravity ("dim"=4, "sdim" = 4).

2 Program. Overall picture

The program consists from the following sections [2]:

"GRAMA" (contents):

PART A

SECTION A1 Function "delprod".

sec. A1a. Definition of "delprod"

sec. A1b. Summation over dummy indices

SECTION A2 Levi-Civita "eps" tensor.

SECTION A3 Spinorial delta-symbol "sdelta".

[2]There are additional subsections of the type A1b.1, A1b.2, etc. in the program itself, which are not shown below in the GRAMA (contents)

PART D

IDENTITIES

The program consists from four files: "progr-a.ma", "progr-b.ma", "progr-c.ma", "progr-id.ma", which are the *Mathematica* scripts. The main text (PART A, SECTIONS A1-A9) is contained in the file "progr-a.ma", the PART B, SECTIONS B1 - B3, are contained in the file "progr-b.ma", PART C is contained in the file "progr-c.ma" and PART D is contained in the file "progr-id.ma". The last script ("progr-id.ma") contains useful identities, which help to simplify expressions, containing Γ-matrices, torsion and curvature superfields.

The order, in which the files are loaded, is crucial for optimal performance of GRAMA. Loading all the scripts "a", "b", "c" and "id" simultaneously, or indiscriminately, may increase the number of computation steps by an order of magnitude. The time of calculation of any specific expression is much reduced, if these files are loaded in sequence, as described below.

First, one loads the "progr-a.ma" script and then the file or expresion which must be calculated (simplified). (If this file/expresion contains matter degrees of freedom, than the script "progr-c.ma" must be loaded simultaneously with "progr-a.ma"). The calculation of the input expression with the help of the "progr-a.ma" (and, if necessary, "progr-c.ma") scripts is the *first step* of the procedure. After it is finished, one should load the file "progr-b.ma" and perform manipulations with the output obtained at the first step. Calculations with the help of the "progr-b.ma" script is the *second step* of the calculation procedure.

Finally, as the *third step*, the output of the second step is simplified with the help of various identities, contained in the "progr-id.ma" script. That provides the final result of the GRAMA calculation procedure.

At each *step* of this procedure one should simplify outputs using operators Expand, SameTerms1, SameTerms2, etc, which are described in the standard *Mathematica* handbook and in the Section 4 of this document.

3 Main physical objects

We define here the correspondence between functions which are used in physical formulas and functions used in the Program.

We are using flat tangent-space vectorial indices, so it is not necessary to fix their position (upper or lower) in the MATHEMATICA notations if the rules of contactions for identical (dummy) indices are defined independently.

In the 16-dimensional formalizm upper and lower spinorial indices are distinguished. We use the notation "Up[x]", for upper spinorial index "x", and "Lo[y]" for lower spinorial index "y". We do not use in the Program an explicit form of matrices, an explicit summation over dummy indices, etc. All operations are defined by their properties as in corresponding theoretical formulas. We use physical notations and sign conventions from [8] (these notations corresponds to that from [9], [10] up to the change of sign of the curvature tensor components). Functions which one considers in GRAMA are superfields on the mass-shell, so their zero-superspace components and zero components of their spinorial derivatives are unambiguously connected with physical fields of D=10 supergravity.

The list of main physical objects is presented in the table. The left column is the MATHEMATICA notation, the right column is the corresponding physical notation.

$delprod[\{a1,\ldots,an\},\{b1,\ldots,bn\}]$	$\delta^{a_1\ldots a_n}_{[b_1\ldots b_n]}$
$gamprod[\{a1,\ldots,an\},\ldots,\{c1,\ldots,cm\},Lo[x],Lo[y]]$	$(\Gamma_{a_1\ldots a_n}\ldots\Gamma^{c_1\ldots c_m})_{xy}$
$gamprod[\{a1,\ldots,an\},\ldots,\{c1,\ldots,cm\},Up[x],Lo[y]]$	$(\Gamma_{a_1\ldots a_n}\ldots\Gamma^{c_1\ldots c_m})^x{}_y$
$eps[\{a1,\ldots,an\}]$	$\epsilon_{a_1\ldots a_n}$
$sdelta[Up[x],Lo[y]]$	$\delta^x{}_y$
$STor[\{a,b\},Up[x]]$	$(T_{ab})^x$
$SAux[a,Lo[x]]$	$(T_a)_x\,;\quad T_a=\Gamma^b\,T_{ab}$
$Tor[\{a,b,c\}]$	T_{abc}
$Tor2[a,b]$	$T_{acd}T_b{}^{cd}$
Tor^2	$T_{abc}T^{abc}$
$SGlu[J,Up[x]]$	$(\lambda^J)^x$
$Glu[\{a,b\},J]$	$\mathcal{F}_{ab}{}^J$
$Rim[\{a,b\},\{c,d\}]$	$\mathcal{R}_{abcd}$
$Ric[a,b]$	$\mathcal{R}_{ab}=\mathcal{R}_{acb}{}^c$
$SRim[a,\{b,c\},Lo[x]]$	$\mathcal{R}_{xabc}$
$vder[f,a]$	$D_a\,f$
$sder[f,Lo[x]]$	$D_x\,f$
Dil	ϕ
$SDil[Lo[x]]$	$\chi_x\equiv D_x\phi$
$meTT[\{a1,\ldots,an\},\{a,b\},\{c,d\}]$	$T_{ab}\,\Gamma_{a_1\ldots a_n}\,T_{cd}$
$meTA[\{a1,\ldots,an\},\{c,d\},b]$	$T_{cd}\,\Gamma_{a_1\ldots a_n}\,T_b$
$meGT[\{a1,\ldots,an\},J,\{c,d\}],\quad etc.$	$\lambda^J\,\Gamma_{a_1\ldots a_n}T_{cd},\quad etc.$
$Asm[\{a1,\ldots,an\}]$	$Asm^{a_1\ldots a_n}$

NOTE!: 1) one must use for vectorial indices the variables start with letters: a,b,c,…,q; spinorial indices start with letters: r,s,…,z and enter as the arguments of functions "Up[ ]" and "Lo[ ]"; Yang-Mills indices start with letters: I,J,K; 2) all objects containing odd number of spinorial indices are fermionic in nature, the product of any pair of fermionic objects X and Y must be represented by the NonCommutiveMultiply function in MATHEMATICA, i.e. $X**Y$. 3) "Asm[{ …}]" is an auxilliary completely antisymmetric function, which is used for contraction of antisymmetric external indices in any expression.

Some comments are presented below which are related with lines of the table.

1) "delprod" is the completely antisymmetric product of δ-symbols. (GRAMA, Sec. A1). The example is:

$$delprod[\{a,b\},\{c,d\}] \equiv \delta^{c,d}_{[a,b]} \equiv \frac{1}{2!}\,(\delta^c_a\,\delta^d_b - \delta^c_b\,\delta^d_a), \tag{3.1}$$

The lists, which are arguments of "delprod", must have equal lengths, not exeeding "dim". The Sec. 1 includes also contraction rules of "delprod" with an arbitrary tensor.

2) "gamprod" is the product of arbitrary number of completely antisymmetric products of γ_a matrices. (GRAMA, Sec. A4). The example is:

$$gamprod[\{a,b\},\{c,d,e\},Lo[x],Lo[y]] \equiv\ (\Gamma_{ab}\,\Gamma_{cde})_{xy} \tag{3.2}$$

where:

$$\Gamma_{ab} \equiv \frac{1}{2!}(\Gamma_a\,\Gamma_b - \Gamma_b\,\Gamma_a) \quad \Gamma_{abc} = \frac{1}{3!}(\Gamma_a\Gamma_b\Gamma_c \pm permutations) \tag{3.3}$$

.

There is the difference between spinorial and vectorial arguments of "gamprod". Vectorial indices are always put in braces ("Lists" in MATHEMATICA notations), this is not the case for spinorial indices. There is an obvious restriction for the number of vectorial indices in each list $Length\{\ldots\} \leq dim$. But the total number of lists is not limited.

The function "gamprod" with odd total number of vectorial indices contains Up-Up or Lo-Lo spinorial indices, but "gamprod" with even total number of vectorial indices contains Up-Lo or Lo-Up spinorial indices.

Symmetry properties of "gamprod" in vectorial and spinorial indices are defined in the GRAMA.

3) "eps"-Levi-Chevita epsilon tensor (GRAMA, Sec. A2). The number of arguments in the function eps[{ ...}] must be equal to "dim".

4) "sdelta"-spinorial delta-symbol (GRAMA, sec. A3).

5) "STor" and "SAux" are the fermionic (spin) torsion-components $(T_{ab})^x$ and $\Gamma^b T_{ab}$ (GRAMA, sec. A7a). T_{ab} is antisymmetric in vector indices and subjected to the constraint: $\Gamma^{ab}\,T_{ab} = 0$ (GRAMA, SECTION B2).

6) "Tor" is the torsion-component T_{abc} (GRAMA, sec. A7b). It is the completely antisymmetric tensor.

7) "SGlu" is the fermionic chiral gluino field $(\lambda^J)^x$ (GRAMA, sec. C1).

8) "Glu" is the gluon field-strength $\mathcal{F}_{ab}{}^J$ which is antisymmetric in vectorial indices (GRAMA, sec. C1).

9) "Rim" is the super-curvature tensor R_{abcd} (GRAMA, sec. A7c). It has usual symmetry properties and is subjected to the constaints:

$$R_{ab}{}^{cd}\,\delta^{ab}_{cd} = \frac{1}{3}\,T^2_{abc} \tag{3.4}$$

.

$$Asm^{abc}\,(R_{abcd} - D_a T_{bcd} - T_{jab} T^j{}_{cd}) = 0 \tag{3.5}$$

$$Asm^{ab} R^j{}_{ab} j = 0 \tag{3.6}$$

$$Asm^{abc}\,(D_a R_{bcij} + T_{ab}\,R_{cij}) = 0 \tag{3.7}$$

(Here and below we do not write spinorial indices explicitely in the cases, where their position may be reconstructed unambiguously).

Function "Ric" is a Ricci tensor. (see GRAMA, sec. A7c).

10) "SRim" is the spinorial components of the corresponding superspace curvature R_{ABCD}. (GRAMA, sec. A7c.). The "SRim" is not independent function:

$$R_{abc} = 2\,\Gamma_{[b}\,T_{c]a} + \frac{3}{2}\Gamma_{[ab}\,T_{c]} \tag{3.8}$$

11) "vder" and "sder" are vectorial and spinorial derivatives (GRAMA, Sec. A8). The action of "sder" is defined according to the following rules. If "sder" appears to the left from "vder" then the GRAMA put it to the right using the commutation relations (see [8] and GRAMA sec. A8d). The example is:

$$\begin{aligned}(D_x\,D_b - D_b\,D_x)\,(V_c)_y = \frac{1}{72}T^{ijk}\,(\Gamma_{ijk}\,\Gamma_b\,D)_x\,(V_c)_y -\\ -(R_{bij})_x\,(\frac{1}{4}\Gamma^{ij}\,V_c)_y - (R_{bcd})_x\,(V^d)_y -\\ -(\Gamma_b\lambda)_x\,(V_c)_y - (V_c)_y\,(\Gamma_b\lambda)_x\end{aligned} \tag{3.9}$$

Here it is supposed that an arbitrary field $(V_c)_y$ is in the algebra of Yang-Mills group G.

The application of "sder" to the main physical superfields is calculated in according to the rules:

$$D\,T_{abc} = 3\,\Gamma_{[a}T_{bc]} + 3\,\Gamma_{[ab}\,T_{c]} \tag{3.10}$$

$$D_x\,(T_{ab})^y = (\hat{O}_{ab})^y{}_x \tag{3.11}$$

where:

$$\hat{O}_{ab} = -\frac{1}{36}\Gamma_{[a}\Gamma^{ijk}D_{b]}\,T_{ijk} + \frac{1}{36\cdot 72}\,\Gamma_{[a}\Gamma^{mnp}\Gamma_{b]}\Gamma^{ijk}\,T_{mnp}T_{ijk}-$$

$$+\frac{1}{72}\,\Gamma^{m}\Gamma^{ijk}T_{abm}T_{ijk} - \frac{1}{4}R_{abij}\Gamma^{ij}$$

and:

$$D_x\lambda^y = \frac{1}{4}\mathcal{F}_{ab}(\Gamma^{ab})_x{}^y \tag{3.12}$$

Furthemore:

$$D\,\mathcal{F}_{ab} = 2\Gamma_{[a}\,D_{b]}\,\lambda - T_{abc}\,\Gamma^c\,\lambda - \frac{1}{36}\,T^{ijk}\Gamma_{ijk}\Gamma_{ab}\,\lambda \tag{3.13}$$

$$DR_{abij} = 2\,D_{[a}\,R_{b]ij} + \frac{1}{36}\,T^{mns}\,\Gamma_{mns}\Gamma_{[a}\,R_{b]ij}+$$

$$+R_{dij}\,T_{ab}{}^d - (\frac{5}{6}\,T_{ijk}\Gamma^k + \frac{1}{36}\,T_{mnp}\Gamma^{mnp}{}_{ij})\,T_{ab} \tag{3.14}$$

12) "meTT", "meTA", "meAA", "meGA", "meGT", "meGG" are matrix elements (see GRAMA SECTIONS B3 and C3.).

The GRAMA (see sec. A7d.) contains also the auxilliary function "Asm". This function is a "quasiconstant" (its vectorial and spinorial derivative is zero, see SEC. A9) and completely antisymmetric in their arguments. It is used in the Program at intrmediate stages for contraction of external indices. It helps to avoid the antisymmetrization procedure in external indices.

4 Main operators

NOTE! , Only the information, related with the "SameTerms1" and "SameTerms2" operators is necessary for User, who is not interested in the internal

structure of the Program. This information is contained in the 4) subsection of this Section

Here we are discussing new operators, which are defined in GRAMA in additional to the standard MATHEMATICA operators.

1) Operator "newexp" (GRAMA, sec.A6c) transforms expressions to the "standard" form. We use the notation "standard" for the expresion, where the symbol " ** " stands inside each pair of fermions, meanwhile any such pair is connected with other pairs, or, with single fermion, or with scalars by means of " * ". For instance (here s,r are any scalars):

$$\text{newexp}[(s * SGlu) * *(r * STor)] = s * r * SGlu * *STor$$

$$\text{newexp}[(s * SGlu) * *(SGlu * r * STor * *STor)] =$$
$$= s * r * SGlu * *SGlu * STor * *STor$$

We are forced to work with the "standard" form, since the symbol " * " means the commutative multiplication and can't stand inside the pair of fermionic functions. Fermions in each pair are automatically transposed according to the anticommutative law so that their spinorial indices are put in lexicographical order (GRAMA, sec.A6a).

2) Operator "vderSimplify" (GRAMA, sec.A8a) serves for pulling quasiconstants "gamprod", "eps", "delprod", etc. from under the symbol "vder". (By definition of a quasiconstant objects, their derivatives are equal to zero). Example:

$$\text{vderSimplify}[vder[gamprod[...] * Tor[a, b, c]]] =$$
$$= gamprod[...] * vder[Tor[a, b, c]]$$

In some cases (for example in commutation relations of spinorial and vectorial derivatives) it is convenient to keep quasiconstants under the symbol of "vder" and to pull them from under this symbol only at the end of calculations with the help of operator "vderSimplify".

3) Operators "MySymm" ("MyAsymm") (GRAMA, sec.A9a) make symmetrisation (antisymmetrisation) of any expression in any indices. For example

$$\text{MySymm}[expression,\ a,b] \quad \text{or} \quad \text{MyAsymm}[expression,\ a,b]$$

produce the expression, symmetric or antisymmetric in indices a,b.

4) Operators "SameTerms1", "SameTerms2" (GRAMA, sec.A9b) serve to simplify any expression. They make identical those terms in an expression, which are equal after rearrangement and redefinition of dummy indices. They differ from each other, and one must use both of them (one after another) in order to identify all equal terms. (The first operator studies an expression identifying identical terms from the left to the right, the second one - from the right to the left).

5) Operator "CountAll" gives the number of all fermions in an expression (GRAMA, sec.A6b).

6) Operator "AllvecIndex" gives the list of all vectorial indices in an expression. Operator "AllspinIndex" gives the list of all spinorial indices in an expression.

Operator "DummyIndex" is applied to the list of indices and gives the list of repeating indices.

All these operators are used as auxiliary in the "SameTerms1", "SameTerms2".

Appendix

Main expansion formulas

We are using here simultaneously the physical notations and notations from GRAMA defined according to the Table from Sec. 3 1. Symplification of products of gamma-matrices is realized by the successive application of expansion formula:

$$\Gamma_{a_1\ldots a_n}\,\Gamma^{b_1\ldots b_m} =$$

$$\sum_{k=0}^{min(m,n)} \eta_k\,(-1)^{k(n+1)}\,k!\,\begin{pmatrix} n \\ k \end{pmatrix}\begin{pmatrix} m \\ k \end{pmatrix}\delta^{[b_1\ldots b_k}_{[a_1\ldots a_k}\ \Gamma_{a_{k+1}\ldots a_n]}{}^{b_{k+1}\ldots b_m]} \qquad (A1)$$

Traces are calculated according to relations:

$$tr\ (\Gamma_{b_1\ldots b_k}\Gamma^{a_1\ldots a_k}) = 16\,k!\,\eta_k\,\delta^{[a_1\ldots a_k]}_{b_1\ldots\, b_k}, \quad k \neq 5$$

$$tr\ (\Gamma_{b_1\ldots b_n}\Gamma^{a_1\ldots a_m}) = 0, \quad n \neq 10-m \qquad (A.2)$$

$$(\Gamma_{a_1\ldots a_5}\Gamma^{b_1\ldots b_5})_\alpha{}^\alpha = 16\cdot 5!\,\delta^{b_1\ldots b_5}_{a_1\ldots a_5} - 16\,\epsilon_{a_1\ldots a_5}{}^{b_1\ldots b_5} \qquad (A.3)$$

$$(\Gamma_{a_1\ldots a_k}\Gamma^{b_1\ldots b_{10-k}})_\alpha{}^\alpha = -16\,\epsilon_{a_1\ldots a_k}{}^{b_1\ldots b_{10-k}} \qquad (A.4)$$

In the preceeding relations the sign before the ϵ-term in the r.h.s. should be changed to + for $(\ )^\alpha{}_\alpha$ position of indices in the l.h.s.

Note! *The "Trace" operator is not used in the program.* The *Trace* is calculated automatically, if User write a matrix with equal upper and lower indices (i.e a matrix of the type gamprod[. . . , Up[x], Lo[x]] or gamprod[. . . , Lo[x], Up[x]]).

In the process of calculations we are using the equation (A1) and other similar equations in the form where all the antisymmetrisators [. . .] are calculated explicitely with the help of the relation:

$$f([a_1\ldots a_k)\,g(a_{k+1}\ldots a_n]) = \sum_{j_k,j_{k-1},\ldots,j_1=1}^{n}$$

$$(-1)^{k(k+1)/2}\,(-1)^{(j_1+\ldots+j_k)}\ \frac{k!(n-k)!}{n!}\,f(a_{j_1}\ldots a_{j_k})\,g(a_1\ldots \dot{a}_{j_1}\ldots \dot{a}_{j_k}\ldots a_n)$$

$$j_k > j_{k-1} > \ldots > j_1\,; \quad n-k \geq k \qquad (A5')$$

$$f([a_1\ldots a_k)\,g(a_{k+1}\ldots a_n]) = \sum_{s_{n-k},s_{n-k-1},\ldots,s_1=1}^{n}(-1)^{k(n-k)}\,(-1)^{(n-k)(n-k+1)/2}$$

$$(-1)^{(s_1+\ldots+s_{n-k})}\ \frac{k!(n-k)!}{n!}\,f(a_1\ldots\dot{a}_{s_1}\ldots\dot{a}_{s_{n-k}}\ldots a_n)\,g(a_{s_1}\ldots a_{s_{n-k}})$$

$$s_{n-k} > s_{n-k-1} > \ldots > s_1\,; \quad n-k \leq k \qquad (A5'')$$

where arguments should be cut away from their places in the cases, when they are dotted, $f(\ldots)$ and $g(\ldots)$ are any completely antisymmetric functions of their arguments (In the case of eq. (A1) these functions play the role of *gamprod* and *delprod*). Eq's (A5') and (A5") help to reduce the $n!$ terms in the antisymmetrizator in the r.h. side of (A1) into the $\frac{n!}{k!(n-k)!}$ terms in the r.h. side of (A5). In the case $n = 2k$ eq's (A5'), (A5") give the same result.

All the formulas (A1)-(A5) (i.e. file "B") are applied *after* the contraction of all dummy indices is fulfilled (this succession helps to decrease the calculation time).

The contaction of indices in the *gamprod*'s is fulfilled with the help of the relation:

$$\begin{aligned} &gamprod[\{a_1 \dots a_k b\}, \{b d_1 \dots d_m\}] = \\ &-\sum_{j=1}^{k} (-1)^{k-j}\ gamprod[\{a_1 \dots \dot{a}_j \dots a_k\}, \{a_j d_1 \dots d_m\}] + \\ &+(d-m)\, gamprod[\{a\}, \{d\}] \end{aligned} \qquad (A6)$$

And in more general case:

$$\begin{aligned} &gamprod[\{a_1 \dots a_k \nu\}, \{b_1 \dots b_n\}, \{c_1 \dots c_m\}, \{\nu d_1 \dots d_p\}] = \\ &= -\sum_{j=1}^{k} (-1)^{k-j}\, gamprod[\{a_1 \dots \dot{a}_j \dots a_k\}, \{b\}, \{c\}, \{a_j d_1 \dots d_p\}] - \\ &-2\,(-1)^n \sum_{j=1}^{n} (-1)^{n-j} gamprod[\{a\}, \{b_1 \dots \dot{b}_j \dots b_n\}, \{c\}, \{b_j d_1 \dots d_p\}] - \\ &-2\,(-1)^{n+m} \sum_{j=1}^{m} (-1)^{m-j} gamprod[\{a\}, \{b\}, \{c_1 \dots \dot{c}_j \dots c_m\}, \{c_j d_1 \dots d_p\}] + \\ &+(-1)^{m+n}(d-p)\, gamprod[\{a\}, \{b\}, \{c\}, \{d\}] \end{aligned} \qquad (A7)$$

where $\{a\} = \{a_1 \dots a_k\},\ \{b\} = \{b_1 \dots b_n\}$, etc.

More general formula, where *gamprod* contains an arbitrary nuber of arguments is a direct generalization of eq. (A7).

The contraction of dummy indices in the *delprod* is realized by the relation:

$$\delta^{[a_1 \dots a_p c_1 \dots c_k]}_{[a_1 \dots a_p d_1 \dots d_k]} = \frac{(dim-k)!\, k!}{(dim-k-p)!\, (k+p)!} \delta^{[c_1 \dots c_k]}_{[d_1 \dots d_k]} \qquad (A8)$$

The action of *delprod* is defined by the relation:

$$\delta^{a_1 \dots a_k a_{k+1} \dots a_n}_{b_1 \dots b_k c_{k+1} \dots c_n} f^{c_{k+1} \dots c_n c_{n+1} \dots c_p} = \delta^{[a_1 \dots a_k}_{b_1 \dots b_k} f^{a_{k+1} \dots a_n]\, c_{n+1} \dots c_p} \qquad (A9)$$

where $f_{c_{k+1} \dots c_p}$ is an arbitrary completely antisymmetric tensor. . The further simplification of the r.h.s. of (A9) must be realized with the help of (A5).

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